Hot-Wire Anemometry in Transonic Flow

C. C. Horstman* and W. C. Rose†
NASA Ames Research Center, Moffett Field, Calif.

The use of hot-wire anemometry for obtaining fluctuating data in transonic flows has been evaluated. From hot-wire heat loss correlations based on previous transonic data, the sensitivity coefficients for velocity, density, and total temperature fluctuations have been calculated for a wide range of test conditions and sensor parameters. For sensor Reynolds number greater than 20 and high sensor overheat ratios, the velocity sensitivity remains independent of Mach number and equal to the density sensitivity. These conclusions were verified by comparisons of predicted sensitivities with those from recent direct calibrations in transonic flows. Based on these results, techniques are presented to obtain meaningful measurements of fluctuating velocity, density, and Reynolds shear stress using hot-wire and hot-film anemometers. Example of these measurements are presented for two transonic boundary layers.

Nomenclature

A'_w = overheat parameters, $1/2[(\partial \ln R_w)/(\partial \ln I)]$
c_p = specific heat at constant pressure
d = wire diameter
E = wire voltage
I = wire current
k = heat conductivity
K = $(d\ln R_w)/(d\ln T_w)$
ℓ = wire length
m = $(d\ln \mu)/(d\ln T)$
M = Mach number
n = $(d\ln k)/(d\ln T)$
Nu = Nusselt number
Pr_t = turbulent Prandtl number
R = resistance
Re = Reynolds number, $\pi u d/\mu$
$R_{(\rho u)T_t}$ = correlation coefficient of mass-flux and total temperature flucturations, $[\overline{(\rho u)' T_t'}]/[<(\rho u)'><T_t'>]$
S = sensor sensitivity coefficent
T = temperature
u, v, w = axial, normal, and spanwise velocity
y = distance normal to wall
α = $1/\{1+[(\gamma-1)/2]M^2\}$
β = $(\gamma-1)M/\{1+[(\gamma-1)/2]M^2\}$
δ = boundary-layer thickness
η = recovery factor, T_r/T_t
μ = viscosity
ρ = density
τ_{wr} = temperature overheat, $(T_w - T_r)/T_r$
$<(\ )>$ = root mean square

Superscripts

$(\ )'$ = fluctuating valve
$\overline{(\ )}$ = time average

Subscripts

e = boundary-layer edge
r = recovery or adiabatic wall
t = total or stagnation conditions
T = temperature
u = velocity
w = wire
ρ = density
ρu = density
ϕ = flow angle

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*Assistant Chief, Experimental Fluid Dynamics Branch. Associate Fellow AIAA.
†Research Scientist. Member AIAA.

Introduction

WITH the rapid advances in computational fluid dynamics over the past few years, computations that were not feasible several years ago now are being performed routinely. These advances, however, have not been followed by advances in the knowledge concerning the physics of complicated fluid flows.[1] One area of current interest is the numerical simulation of transonic flow about aerodynamic bodies. Recent transonic flow calculations[2] have shown that the turbulence model employed strongly affects the calculated flowfield and that none of the existing models adequately predicts the experimental results. To date, the development of new turbulence models has relied on intuition and measured mean flow data. The direct measurement of the required turbulence quantities would provide data that could be used for the development of improved turbulence models and provide additional insight into the physics of turbulence. Unfortunately, fluctuating turbulence data are virtually nonexistent for transonic flows.

In principle, the hot-wire anemometer can be used to obtain fluctuating velocity, density, and shear stress measurements in transonic flows. However, hot-wire measurements have not been exploited in transonic flows for several reasons. These include the difficulties in determining accurate sensitivity coefficients as well as wire breakage, vibration, and strain gaging problems associated with the high dynamic pressures incurred at transonic flows. The principle problem in determining accurate sensitivity coefficients is that the velocity and density sensitivity coefficients vary with Mach number and, in general, are not equal.[3,4] Previous calculations,[3] based on calibration data available at the time, indicated that the velocity sensitivity coefficients varied by an order of magnitude with small changes in Mach number. This makes sensor calibrations extremely difficult to obtain and, since the density and velocity sensitivity coefficients are not equal, modal analysis techniques cannot be used to resolve the desired flow quantities.

The present paper reevaluates the use of hot-wire anemometry for obtaining fluctuating data in transonic flows in light of the recent hot-wire sensitivity calibrations of Rose and McDaid[5] and Mateer et al.[6] and the recent developments of hot-wire sensors[5,7] and anemometer systems. The conclusion is that the previously mentioned problems can be overcome and meaningful fluctuating measurements can be obtained in transonic flows. By use of Behrens' hot-wire response correlations[8] with appropriate end loss corrections following Dewey[9] to obtain hot-wire sensitivities (validated by the direct calibration results of Refs. 5 and 6), a parametric study is conducted to define a range of Reynolds numbers and hot-wire overheat ratios, where the velocity sensitivity coefficients are independent of Mach number and equal to the density sensitivity coefficients. In order to determine a range of wire overheat ratios where the mass-flux fluctuations can be measured directly, in lieu of their being obtained by modal analysis techniques, the relative magnitudes of the total temperature and velocity or density sensitivity coefficients are also investigated. The relations required to deduce the fluctuating velocity, density, and shear stress from the measured quantities are then presented. Several types of the sensors described eliminate wire breakage, vibration, and strain gaging problems. Although these sensors cannot be used with constant current anemometers (these anemometers can only be used with sensors with well-defined time constants), commercial constant temperature anemometers are now available with adequate frequency response for transonic flows. Finally, several examples of velocity and density fluctuations and shear stress measurements obtained in transonic boundary layers are presented.

Discussion

Hot-Wire Sensitivities

General Equations

Following the principles set forth by Morkovin[3] and Kovasznay,[4] the fluctuating voltage of a heated wire held normal to the flow can be expressed in terms of the fluctuations in velocity, density, and total temperature as

$$\frac{E'}{\bar{E}} = S_\rho \frac{\rho'}{\bar{\rho}} + S_u \frac{u'}{\bar{u}} - S_{T_t} \frac{T_t'}{\bar{T}_t} \tag{1}$$

where the sensitivity coefficients are defined for constant temperature operation following Morkovin's terminology,[3] as

$$S_\rho = \frac{1}{2}\left(\frac{\partial\, \ell n Nu_t}{\partial\, \ell n Re_t} - \frac{1}{\tau_{wr}} \frac{\partial\, \ell n \eta}{\partial\, \ell n Re_t}\right) \tag{2}$$

$$S_u = S_\rho + \frac{1}{2\alpha}\left(\frac{\partial\, \ell n N_t}{\partial\, \ell n M} - \frac{1}{\tau_{wr}} \frac{\partial\, \ell n \eta}{\partial\, \ell n M}\right) \tag{3}$$

$$S_{T_t} = \frac{K}{2A'_w} + \frac{1}{2}(K - 1 - n_t) + m_t S_\rho + \frac{1}{2}(S_u - S_\rho) \tag{4}$$

According to Morkovin,[3] for high-speed flows ($M > 1.2$), $S_u = S_\rho$ and Eq. (1) can be written as

$$\frac{E'}{\bar{E}} = S_{\rho u} \frac{(\rho u)'}{\overline{\rho u}} - S_{T_t} \frac{T_t'}{\bar{T}_t} \tag{5}$$

where $S_{\rho u} = S_\rho$. By use of the modal analysis techniques of Kovasznay[4] to solve Eq. (5), we can obtain the fluctuating physical variables $\langle(\rho u)'\rangle$, $\langle T_t'\rangle$, and $R_{(\rho u)T_t}$ by operating the wire at different overheat ratios.[10-14] However, for transonic flow, the derivatives of the Nusselt number and recovery factor with Mach number are not zero, and in general $S_u \neq S_\rho$. In fact, calculations given by Morkovin,[3] based on hot-wire sensitivity data available at that time, indicate that the term $(\partial \ell n Nu_t)/(\partial \ell n M)$ is very large in the transonic flow regime at all wire overheats for a wide range of Reynolds numbers. This prohibits the use of the modal analysis technique, and probably has discouraged the use of hot-wire anemometry in transonic flows for the past 20 years. In principle, if S_u and $S\rho$ could be determined for a hot-wire sensor, then, by operating at six overheat ratios, the fluctuating quantities $\langle\rho'\rangle$, $\langle u'\rangle$, and $\langle T_t'\rangle$ and their cross-correlations could be obtained by inversion of a 6×6 matrix. This has been attempted[5,15] but without success, since the variation of S_u and S_ρ with wire temperature is small, and the resulting matrix is nearly singular. Although it is certainly desirable to obtain separate measurements of $\langle\rho'\rangle$ and $\langle u'\rangle$, it is doubtful that they can be obtained until better techniques are developed for measuring more precise values of S_u and S_ρ.

Direct Calibration Measurements

A recent investigation by Rose and McDaid,[5] where the hot-wire sensitivity coefficients S_ρ and S_u were measured directly by varying density and velocity independently in the transonic flow regime, has shown that the ratio S_u/S_ρ is rather insensitive to changes in overheat, if the overheat is high. These newer data are summarized in Figs. 1-3. The measurements were obtained using a specially designed 5-μm tungsten wire over a Mach number range from 0.3 to 1.2 and a wire Reynolds number range from 20 to 400. Within the accuracy of the measurements ($\pm 10\%$), the resulting sensitivity coefficients were found to be essentially independent of Mach and Reynolds numbers. These results are in apparent contradiction with previous predictions.[3]

Subsequent to Morkovin's calculations,[3] a significant amount of additional transonic hot-wire calibration data has been obtained. Behrens[8] has correlated these data successfully (including the measurements of Vrebalovich,[16] Christiansen,[17] and Laufer and McClellan[18]) in the form of recovery factor and Nusselt number as functions of Mach and Reynolds numbers. By use of these correlations, we evaluated the sensitivity equations for S_u and S_ρ [Eqs. (2) and (3)] for the test conditions of Rose and McDaid.[5] The correlations were modified to account for finite wire lengths, using the end-loss correlations proprosed by Dewey.[9] The resulting values for S_ρ and S_u are compared with the data on Figs. 1-3 for three-wire length-to-diameter ratios at a wire Reynolds number and Mach number representing average values for the experimental data ($Re_t = 100$, $M = 0.8$). The actual wire ℓ/d was measured to be approximately 75. Comparing the experimental and calculated results, we see that the trends of the data and the predictions agree. The magnitude of the predicted sensitivities vary over the experimental Reynolds number range, but show a negligible variation with Mach number for overheat ratios greater than 0.2. The calculated ratio S_u/S_ρ (Fig. 3) is in excellent agreement with the experimental data; the plotted values show that the two sensitivities are essentially equal for temperature overheat ratios greater than 0.4. It also was determined that the ratio S_u/S_ρ varies slowly with Re_t, M, and ℓ/d for the range of experimental values (represented by the extremes on the bars on Fig. 3). In order to predict adequately S_ρ and S_u alone, direct calibration methods must be used, since the effective ℓ/d of any sensor is a function of many variables and, therefore, may never be known adequately for predictive purposes.

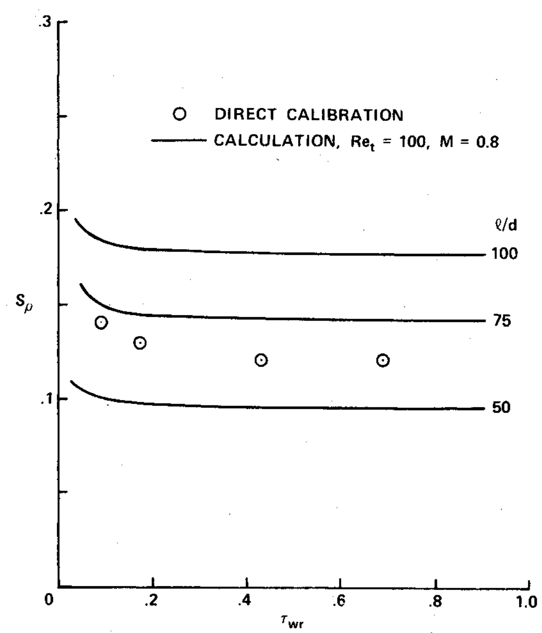

Fig. 1 Comparison of calculated and measured density sensitivities.

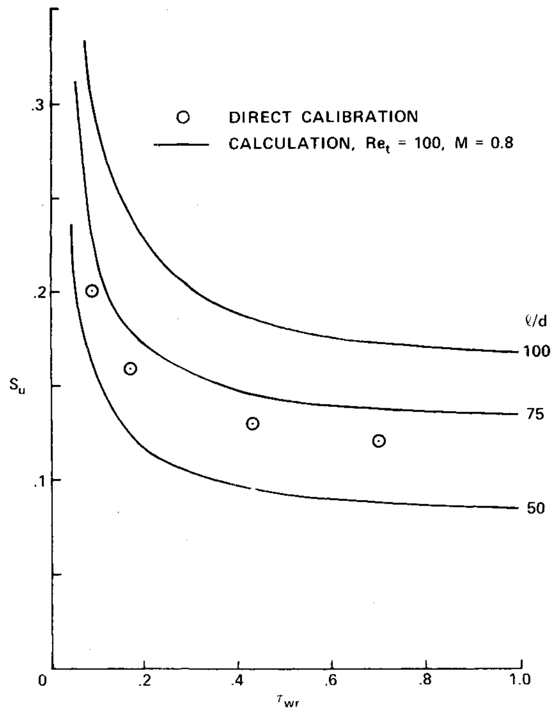

Fig. 2 Comparison of calculated and measured velocity sensitivities.

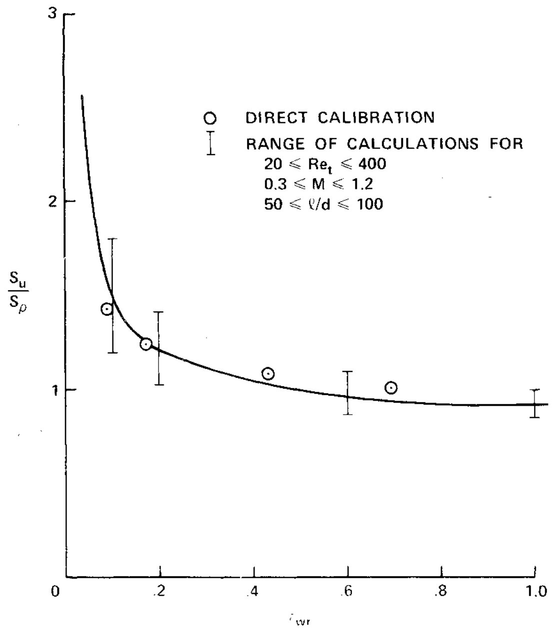

Fig. 3 Comparison of calculated and measured velocity-density sensitivity ratios.

Additional evidence that Behrens' correlations can be used to predict the trends in recent calibration data is shown in Fig. 4, where the calculated sensitivities are compared with the experimental results from Ref. 6. The measurements were obtained by traversing a transonic boundary layer with a 20-μm-diam commercial film sensor. The flow conditions varied from $M=0.9$ at $Re_t=180$ near the wall to $M=1.4$ at $Re_t=330$ at the outer edge of the boundary layer; it was assumed the sensor was responding to $S_{\rho u}$ (i.e., $S_u=S_\rho$). If S_u/S_ρ were a function of Mach number, one would expect significant variations of the measured sensitivity coefficient across the boundary layer. This was not the case. The data are compared with the present prediction method, and excellent agreement is obtained in the sense that the predicted values of S_u and S_ρ are almost equal and independent of Mach and Reynolds numbers for the range of experimental data. The extremes on the bars represent the effect of varying Mach number from 0.9 to 1.4. (The small differences between the two calculated sensitivity coefficients are within the error limits one could expect from measured compressible turbulence data.) For these calculations, it was assumed that the sensor had no end losses, a reasonable assumption for film sensors. In this case, not only the trends of the data are predicted, but the absolute magnitude of the data also are predicted within 20%. By comparing the calibration results from the film sensor with the hot-wire correlations, we have assumed that the hot-film sensitivity is essentially the same as that of a hot-wire. Detailed measurements at Ames[6] have shown this assumption to be valid for compressible flows.

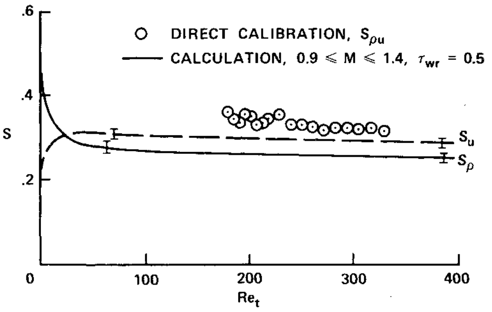

Fig. 4 Comparison of calculated density and velocity sensitivities and measured mass-flux sensitivity.

The apparent contradiction between the calibration results discussed in the preceding and the previous predictions by Morkovin[3] is due to Morkovin's estimate (for predicting S_u) of the term $\partial \ln Nu_t/\partial \ln M$ in Eq. (3). The current results[5,6] clearly show, from direct measurements of wire voltage vs the mass flux ρu through the transonic Mach number range, that the term $\partial \ln Nu_t/\partial \ln M$ is extremely small for many test conditions. Differentiation of these data[5,6] was relatively simple, since almost continuous variations of mass flux and Mach number were obtained. Morkovin had the disadvantage of trying to differentiate individual data points between relatively large differences in Mach number. On the other hand, the more recent correlation by Behrens[8] is based on a significant amount of additional data that predict the same transonic behavior as the current results.[5,6]

Parametric investigation of S_u/S_ρ

The previous comparisons (Figs. 1-4) show that the trends in some of the recent transonic calibration data can be predicted. It is proposed that this calculation technique can be used to define a domain of hot-wire and flow variables where $S_u \approx S_\rho$. By testing within this domain, meaningful fluctuating measurements can be obtained at transonic speeds.

The sensor chosen as a baseline for the calculations was a tungsten wire, with an $\ell/d=100$ (similar to the probe used in Ref. 5). The calculated ratio S_u/S_ρ for the baseline sensor, plotted as a function of wire Reynolds number and temperature overheat ratio, is shown on Fig. 5 for $M=1.0$. For wire Reynolds numbers greater than 20 and temperature overheat ratios greater than 0.5, the sensitivity ratio is close to unity. For low Reynolds numbers, there appears to be no overheat ratio for which the velocity and density sensitivities are equal.

The variation of the calculated sensitivity ratio with Mach number is shown in Figs. 6 and 7. For a Reynolds number equal to one (Fig. 6), the sensitivity ratio is only close to one for high Mach numbers, Reynolds numbers, and overheat ratios. At a Reynolds number equal to 100 (Fig. 7), the sensitivity ratio is approximately equal to 1 over the entire Mach number range, if $\tau_{wr}>0.5$. At low overheats, the ratio reaches a maximum near $M=1.1$, but for $M<0.4$ or >2.5, the ratio is close to 1 over the entire range of wire overheat ratios.

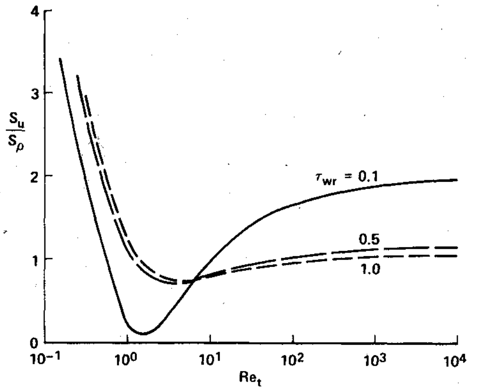

Fig. 5 Variation of velocity-density sensitivity ratio with Reynolds number and temperature overheat at $M = 1.0$ and $\ell/d = 100$ for tungsten wire.

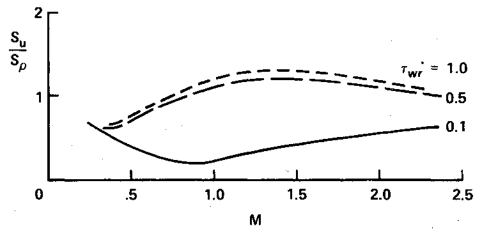

Fig. 6 Variation of velocity-density sensitivity ratio with Mach number and temperature overheat at $Re_t = 1.0$ and $\ell/d = 100$ for tungsten wire.

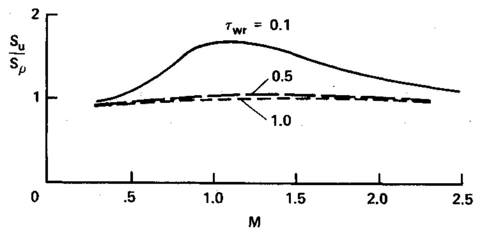

Fig. 7 Variation of velocity-density sensitivity ratio with Mach number and temperature overheat at $Re_t = 100$ and $\ell/d = 100$ for tungsten wire.

The sensitivity ratio also has been calculated for various sensor materials and length-to-diameter ratios. These results are summarized in Figs. 8 and 9. Sensor material and ℓ/d do not influence the results for high values of Re_t, but, for low values of Re_t, their influence is significant (because of large end-loss corrections), and there is no region where $S_u \approx S_\rho$.

In summary, these calculations indicate that, for wire-temperature overheat ratios greater than 0.5 and Reynolds numbers greater than 20, one can assume $S_u \approx S_\rho \approx S_{\rho u}$ for transonic Mach numbers (an assumption previously demonstrated[3,18] for Mach numbers greater than 1.2). However, no range of test conditions seems to exist where $S_u \approx S_\rho$ for all overheat ratios in the transonic Mach number range. Therefore, when applying the modal analysis technique to determine the mass-flux and total temperature flucturations, only data at overheat ratios greater than 0.5 can be used. Since $\langle T_t' \rangle$ and $R_{(\rho u) T_t}$ are essentially determined by the low overheat data, the extrapolation of the data obtained at overheats greater than 0.5 to low overheat values may cause very large errors.

Evaluation of $\langle (\rho u') \rangle$ and $\langle T_t' \rangle$

As was previously mentioned, the modal analysis technique can be employed at transonic speeds to determine $\langle (\rho u)' \rangle$ and, with questionable accuracy, $\langle T_t' \rangle$. For many adiabatic shear flows of aerodynamic interest, the measured mean total

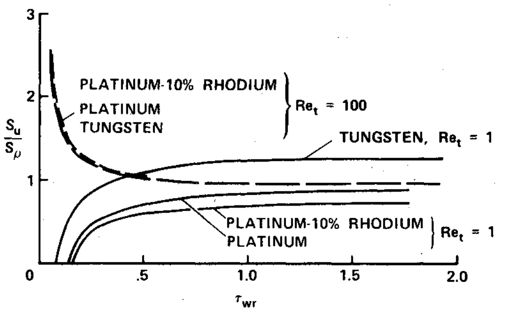

Fig. 8 Variation of velocity-density sensitivity ratio with wire material and temperature overheat at $M = 1.0$, $\ell/d = 100$, and $Re_t = 1$ and 100.

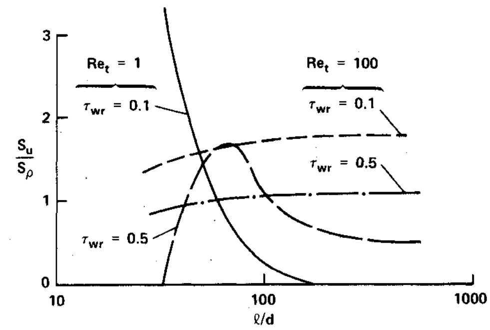

Fig. 9 Variation of velocity-density sensitivity ratio with ℓ/d and temperature overheat at $M = 1.0$ and $Re_t = 1$ and 100 for tungsten wire.

temperature gradients are negligible, and one usually can assume that the total temperature flucturations also are negligible. Thus, the mean-flux fluctuations can be measured directly. Actually the terms to be compared to neglect one fluctuation component is the product of the fluctuation and its sensitivity coefficient [Eq. (5)], and not the fluctuation alone. Therefore, care must be taken to insure that the wire is operated at overheats where the sensitivity coefficients are of the same order. This assumption could be verified by operating the sensor at two overheat ratios (both high enough to insure $S_u = S_\rho$) and evaluating the mass-flux fluctuations for each overheat, assuming zero total temperature fluctuations. If the two values agree, the assumption is valid, since the senor sensitivity to total temperature is a function of overheat ratio.

If the total temperature fluctuations are not negligible, care must be taken to operate the sensor at overheat ratios where $S_{T_t} \gg S_{\rho u}$ or $S_{\rho u} \gg S_{T_t}$ when measuring total temperature or mass-flux flucturations, respectively. Using Eqs. (2) and (4), the ratio S_{T_t}/S_ρ has been calculated for a series of wire Reynolds numbers and wire materials, and is shown as a function of overheat ratio on Figs. 10 and 11. [In order to evaluate A_w' in Eq. (4), any possible variation in Nusselt number with overheat ratio was neglected. Although A_w' should be measured directly for each flow measurement, for most test conditions, the resulting error in S_{T_t} due to this assumption is small.[3,4,10,12]] The results show that, for a sensor to be insensitive to total temperature fluctuations, the wire temperature overheat ratio must be greater than 1.5. For room temperature air (300°K), this requires a wire operating temperature greater than 750°K. These results are essentially independent of Re_t and sensor material and, although not shown, independent of Mach number and ℓ/d. At low overheats, the sensors predominantly are sensitive to total temperature fluctuations.

In order to measure $\langle T_t' \rangle$ directly, a constant current anemometer must be used because of frequency response limitations of constant temperature anemometers at low overheat ratios.[12] For the direct measurement of $\langle (\rho u)' \rangle$ at high overheats, either system can be used. Although Eqs. (2-4)

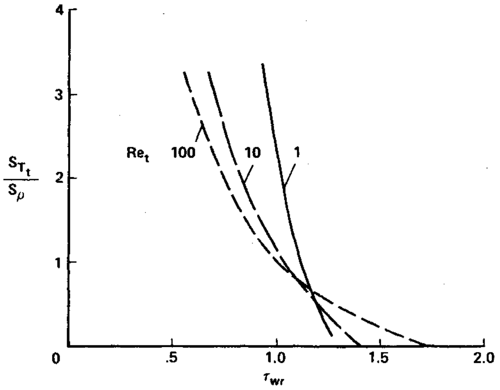

Fig. 10 Variation of total temperature-density sensitivity ratio with temperature overheat and Reynolds numbers at $M=1.0$ and $\ell/d=100$ for tungsten wire.

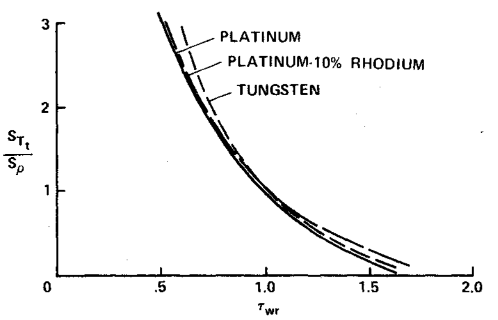

Fig. 11 Variation of total temperature-density sensitivity ratio with temperature overheat and wire material at $M=1.0$, $Re_t=100$, and $\ell/d=100$.

are valid only for constant temperature operation, the calculated ratios S_u/S_ρ and S_{T_t}/S_ρ are valid for either system.

Equations for $\langle u'\rangle$, $\langle\rho'\rangle$, and $\bar\rho\,\overline{u'v'}$

Since the fluctuating quantities obtained from hot-wire measurements are $(\rho u)'$, v', T_t' and their cross-correlations, various assumptions must be employed to deduce the velocity and density fluctuations and Reynolds shear stress from the measured quantities. In order to calculate $\langle u'\rangle$ and $\langle\rho'\rangle$ from normal wire measurements of $\langle(\rho u)'\rangle$ and $\langle T_t'\rangle$, an assumption must be made concerning the fluctuating pressure and its correlations with velocity and temperature. The usual assumption is that $\langle p'\rangle$ is negligible. This assumption has been discussed at length[5,10,12-14] and should be valid for most transonic flows. The resulting equations[3] for $\langle u'\rangle$ and $\langle\rho'\rangle$ are

$$\frac{\langle u'\rangle}{\bar u} = \frac{1}{\alpha+\beta}\left[\alpha^2\left(\frac{\langle(\rho u)'\rangle}{\overline{\rho u}}\right)^2 + \left(\frac{\langle T_t'\rangle}{\bar T_t}\right)^2 + 2\alpha\frac{\langle(\rho u)'\rangle}{\overline{\rho u}}\,\frac{\langle T_t'\rangle}{\bar T_t}R_{(\rho u)T_t}\right]^{1/2} \tag{6}$$

$$\frac{\langle\rho'\rangle}{\bar\rho} = \frac{\langle T'\rangle}{\bar T} = \frac{1}{\alpha+\beta}\left[\beta^2\left(\frac{\langle(\rho u)'\rangle}{\overline{\rho u}}\right)^2 + \left(\frac{\langle T_t'\rangle}{\bar T_t}\right)^2 - 2\beta\frac{\langle(\rho u)'\rangle}{\overline{\rho u}}\,\frac{\langle T_t'\rangle}{\bar T_t}\times R_{(\rho u)T_t}\right]^{1/2} \tag{7}$$

Since modal analysis techniques are invalid at low overheats in transonic flows, direct measurement techniques should be employed to determine $\langle T_t'\rangle$ and $R_{(\rho u)T_t}$. Direct measurements of $R_{(\rho u)T_t}$ have been obtained for supersonic flows, and are described in Ref. 7. For cases in which both the total temperature and pressure fluctuations are negligible, a single voltage reading at high overheat will yield both the normal velocity and density fluctuations.

In order to obtain the Reynolds shear stress from hot-wire measurements, the assumption that the pressure vertical-velocity correlation $\overline{p'v'}$ is negligible must be made.[7,10] This assumption is less restrictive than that required for calculating $\langle u'\rangle$ or $\langle\rho'\rangle$, since $\overline{p'v'}$ can be negligible, although $\langle p'\rangle$ is not. The resulting expression[7] for shear stress is

$$\bar\rho\,\overline{u'v'} = \frac{C_p\bar T}{C_p\bar T+\bar u^2}\left(\overline{(\rho u)'v'} + \frac{\bar u\,\bar\rho}{\bar T}\overline{T_t'v'}\right) \tag{8}$$

In order to measure $\overline{(\rho u)'v'}$ and $\overline{T_t'v'}$, a sensor inclined to the flow must be used.[7,10,19] Techniques using dual and triple sensor probes for the direct measurement of these quantities, which must be employed in transonic flows, are described in Ref. 7. An additional sensitivity coefficient (the sensitivity to flow angle) also must be determined. If the sensor is aligned 45° to the mean-flow direction, the sensitivity to flow angle is approximately equal to $S_{\rho u}$ for high overheats.[7,19] The angle sensitivity of a particular sensor also can be measured directly by pitching the sensor in the flow.[10,19]

Since the measurement of $\overline{T_t'v'}$ requires a special probe design,[7] an alternate equation [Eq. (9)] for obtaining Reynolds shear stress has been proposed.[20] The equation, which makes the additional assumption of a known turbulent Prandtl number, is

$$\bar\rho\overline{u'v'} = \left[1+\frac{(\gamma-1)M^2}{Pr_t}\left(1-\frac{C_p\,d\bar T_t/dy}{\bar u d\bar u/dy}\right)\right]^{-1}\overline{(\rho u)'v'} \tag{9}$$

For adiabatic flow, this relation reduces to

$$\bar\rho\,\overline{u'v'} = \left[1+\frac{(\gamma-1)M^2}{Pr_t}\right]^{-1}\overline{(\rho u)'v'} \tag{10}$$

Note that this relationship requires no knowledge of the total temperature fluctuations.

Previous measurements[21-23] in subsonic and supersonic boundary layers have shown that the turbulent Prandtl number varies from 1.0 to 0.7 across a boundary layer, except when close to the wall and at the outer edge. Equations (9) and (10) show that, as the Mach number increases, the Prandtl number assumption becomes more critical. For adiabatic flow, a 10% error in Prandtl number results in a 1% error in shear stress at Mach 0.5, a 3% error at Mach 1.0, and a 6% error at Mach 2.0. Considering the possible errors involved in obtaining transonic fluctuating data, these errors due to the Prandtl number assumption are not considered significant.

Sensors

Because of the high dynamic pressures incurred at transonic flows, wire breakage can be a severe problem. Rose and McDaid[5] have been successful in constructing tungsten wire probes, which have relatively small length-to-diameter ratios and which survive in high Reynolds number transonic flows. Recall that ℓ/d has little effect on the ratio of sensitivities. An alternate solution to the wire breakage problem is to use specially constructed probes with the wires mounted on high-temperature ceramic wedges.[7] For some flows, the wire diameter that is required to insure that the wire Reynolds number is greater than 20 may be too large to obtain adequate frequency response. In this case, a commercial film probe could be used.

Since both the ceramic wedge and film sensors do not have well-defined time constants, constant temperature anemometers must be used to power these sensors. New commercial anemometers now are available with a special film bridge circuit which, at high overheat, attains a flat frequency response up to 150 kHz.[7] This frequency response should be adequate for most experimental test flows at transonic speeds.

For inclined sensors, several additional complications must be considered. The principle problems are wire vibration and strain gaging effects which, when present, prohibit accurate measurement of the fluctuating vertical velocity and shear stress. To eliminate these problems, either wires mounted with substantial slack or wedge probes can be employed. However, both of these solutions also offer certain disadvantages. For inclined wires with slack, the sensitivity to flow angle may become too small and inaccurate to obtain meaningful measurements. For wedge probes, especially those employing more than one sensor, thermal feedback problems can cause the probe sensitivities to be functions of frequency.[24] This makes standard static calibration techniques invalid. For frequencies above 200 Hz, these effects are no longer present for most sensors, and the sensitivities are independent of frequency. Since, for the most high-speed flows, only a small percentage of the total turbulent energy is contained in this low-frequency interval,[6,7,10,14] dynamic calibration methods (calibrating the probe in a flowfield previously measured with a single sensor) can be used to determine the probe sensitivities. Knowing these sensitivities, one can obtain quantitative measurements by neglecting the small errors due to the change in sensitivity at low frequency. As an alternative to calibrating the inclined sensor, one can use a dual sensor to determine the ratio of the vertical-velocity to mass-flx fluctuations and the vertical velocity mass-flux correlation coefficient. These measurements are independent of the magnitude of the sensitivity coefficients and only require knowledge of the ratio $S_\phi / S_{\rho u}$. The mass-flux fluctuations then can be determined with a single normal sensor.

Additional problems that occur when using crossed- or dual-wedge sensors include unequal sensitivities, spatial resolution, and mean flow and turbulence gradient effects. These effects are discussed by Sanborn.[19]

Transonic Flow Measurements

Fluctuating measurements are presented for two boundary layers with edge Mach numbers approximately equal to 0.8. The first set of measurements[5] was obtained on a flat plate at $M_e = 0.8$, $\delta = 9$ cm, and $Re_\delta \approx 1.0 \times 10^6$. A normal tungsten wire was used to obtain $\langle (\rho u)' \rangle$. The second set of measurements was obtained by Mikulla (private communication) on the side wall of a pilot version of the Ames High Reynolds Number Channel at $M_e = 0.78$, $\delta = 3$ cm, and $Re_\delta \approx 0.4 \times 10^6$. Both single and dual commercial-film wedge sensors were used to obtain $\langle (\rho u)' \rangle$, $\langle v' \rangle$, $\langle w' \rangle$, and $\overline{(\rho u)' v'}$. A summary of the fluctuating intensities is given in Fig. 12. The sensors for both flows were operated at temperature overheat ratios of 0.7. This insured that $S_u = S_\rho = S_{\rho u}$. The rms velocity and density fluctuations were calculated by using Eqs. (6) and (7) and by assuming negligible total temperature fluctuations. Note the excellent agreement between the measurements obtained in the two

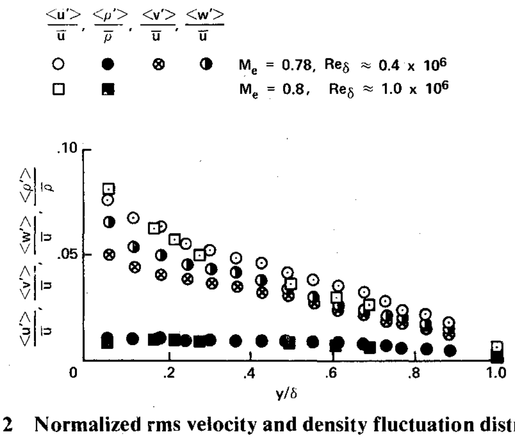

Fig. 12 Normalized rms velocity and density fluctuation distribution across the boundary layer.

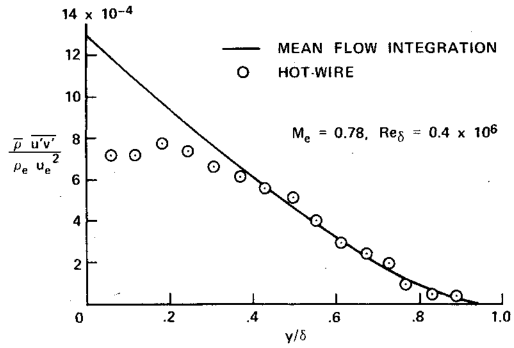

Fig. 13 Normalized Reynolds shear stress distribution across the boundary layer.

flows. The lateral and vertical velocity fluctuations were obtained by using the dual film sensor to determine the ratios of lateral and vertical velocity to mass-flux fluctuations.

Reynolds shear stress measurements obtained for the Mach 0.78 flow are presented in Fig. 13. Equation (10) was used to calculate the shear stress with $Pr_t = 0.9$. The data are compaed with integrated values obtained by using the method of Ref. 20, from measured values of mean velocity, mean temperature, and wall shear. The data are in excellent agreement with the integrated value for $y/\delta > 0.25$, which validates the Prandtl number and negligible pressure vertical-velocity correlation assumptions for the outer portion of this particular flow. Near the wall, the measurements are low, because of spatial resolution and interference effects which influence the data significantly in regions where the normal velocity gradients are large.[19] The probe size was approximately 15% of the boundary-layer thickness. Smaller probes could be expected to minimize these effects.

Conclusion

The use of hot-wire anemometry for obtaining fluctuating data has been evaluated for transonic flows. Using hot-wire sensitivity correlations developed by Behrens[8] (validated by recent sensor calibration measurements), we calculated the density, velocity, and total temperature sensitivities for a wide range of sensor and flow variables. These results show that, for sensor Reynolds numbers above 20 and high sensor temperature overheat ratios, the velocity sensitivity coefficients are essentially independent of Mach number and equal to the density sensitivity coefficients. Therefore, direct measurements of the mass-flux fluctuations can be obtained, if the total temperature fluctuations are negligible, or the sensors are operated at overheat ratios sufficiently high to eliminate their sensitivity to total temperature fluctuations. Sensor calibration procedures also are simplified, since the sensitivity to density is obtained easily by placing the sensor in the freestream and varying the wind-tunnel total pressure. From the measured mass-flux and vertical-velocity fluctuations, the fluctuating density and longitudinal velocity intensitites and Reynolds shear stress can be deduced by using assumptions appropriate to most transonic flows. Examples of these measurements for two boundary layers, presented herein, show the feasibility of hot-wire anemometry in transonic flows.

The ultimate use for transonic shear-layer turbulence measurements would be for the validation of existing turbulence models and the provision of data to develop new models. As has been shown, in order to obtain these measurements in transonic flow, several assumptions must be made. These assumptions, together with calibration uncertainties, limit the accuracy of most transonic fluctuating data to ±10%. However, these accuracies are acceptable in view of the very large differences between many proposed turbulence models.

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